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TOPOLOGY OF 2D BOUNDARY LAYER ERUPTION BASED ON DIFFERENT VORTEX CRITERIA

A. R. NIELSEN[1]†, M. HEIL[2], M. ANDERSEN[3] AND M. BRØNS[1]
[1]Department of Applied Mathematics and Computer Science, Technical University of Denmark.
[2]School of Mathematics, University of Manchester.
[3]Department of Science, Systems and Models, Roskilde University
†anry@dtu.dk

The notion of a vortex is central in fluid dynamics, it is however a longstanding question how we define a vortex mathematically. A widely used method to identify vortices is the Q-criterion by Hunt et al. (1988), which for a 2D flow reduces to $Q = u_x v_y - u_y v_x > 0$. In this study we focus on identifying what happens when vortices are created or destructed. This is done by developing a bifurcation theory for the contour curves where $Q = 0$. We show that four possible types of bifurcations occur under specific conditions. These correspond to a vortex region occurring on the boundary, erupting from the boundary, occurring away from the boundary or a vortex region separating into two vortex regions. For further details and proofs, please refer to Nielsen (2017). The topological changes in the vortex structure depends on the chosen vortex criterion. To investigate this dependency we compare the Q-criterion with a definition of vortices where a local extremum of vorticity is defined as a feature point for a vortex. Under generic assumptions on higher-order derivatives of ω it can be shown (Brøns 2007) that the creation of a vortex corresponds to a topological saddle-center bifurcation of the vorticity field.

(a) (b)

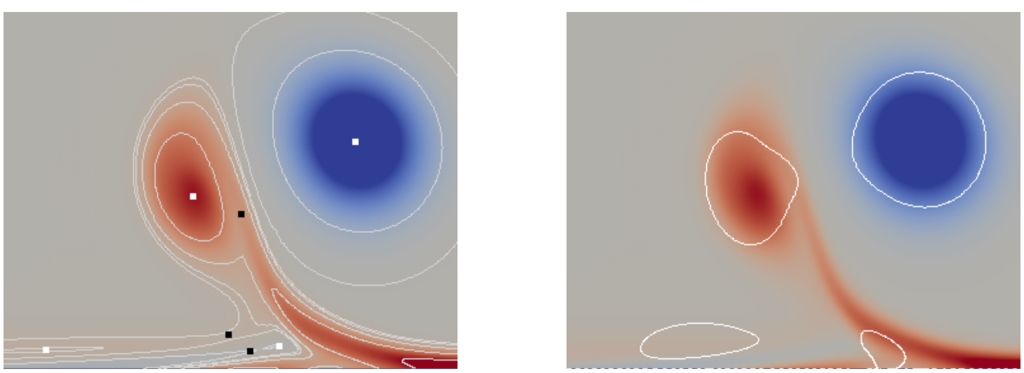

Figure 1: Numerical results of the eruption problem for $Re = 1500$ and $t = 67$. The vorticity contours with values from -0.5 to 0.5 are colored in blue and red. In (a) the vortices are identified by white squares corresponding to a local extremum of the vorticity. The black squares show the location of saddle points, while the gray lines represent some selected contours of the vorticity. In (b) the vortices are identified as the regions with positive Q values. These regions are encircled by the white lines which corresponds to the zero contour curves of Q.

As an application we consider the well known boundary layer eruption phenomenon. The phenomenon is studied in comparison with earlier studies by Andersen (2013) and Kudela & Malecha (2009), where an initial Gaussian distributed vorticity patch is convected close to a no-slip boundary. Our numerical simulations are preformed using the open source finite element library `oomph-lib` which is developed by Heil and Hazel (2006). In Figure 1 the vortices are identified by the two different criteria at the same state of the flow. This example indicates that there is not a clear alignment between the two definitions. Even though the number of vortices is the same the position differs slightly. In order to give a complete overview of the initial phase of the eruption process, we make a bifurcation analysis based on the two different criteria. In Figure 2 our numerical results are presented in the form of two bifurcation diagrams. For both criteria all the observed changes in topology are marked as lines in the (t, Re)-space. These bifurcation diagrams show that in general there is no simple correlation between our two choices of vortex criteria. There are, however, some similarities. In both cases we observe a more complex vortex structure for higher Re, and for some of the observed bifurcations the time where the bifurcation occur tend to be independent of Re for higher values Re. These observations lead us to the conclusion that an analysis of a vortex formation may be highly dependent on the chosen definition of a vortex.

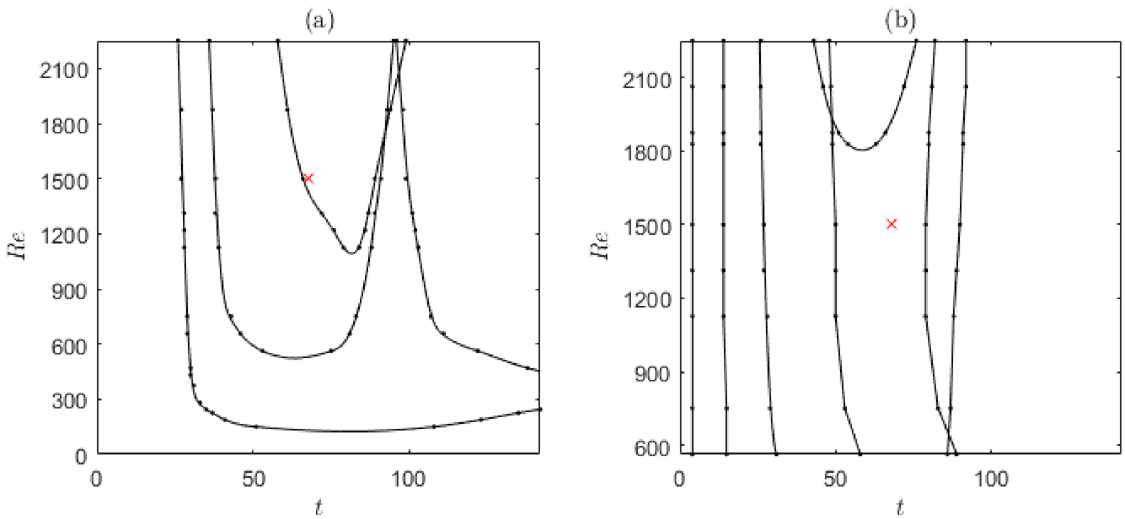

Figure 2: bifurcation diagrams of the eruption problem. (a) is based on the vorticity extremum criterion and (b) is based on the Q criterion. The red crosses correspond to the examples shown in Figure 1.

References

1 M. HEIL AND A. L. HAZEL: oomph-lib: An Object-Oriented Multi-Physics Finite-Element Library. In: Fluid-Structure Interaction, Editors: M. Schafer and H.-J. Bungartz. Springer (Lecture Notes on Computational Science and Engineering), pp. 19-49, (2006). oomph-lib is available as open-source software at http://www.oomph-lib.org.

2 A. A. WRAY, P.MOIN AND J. C. R. HUNT: Eddies, streams and convergence zones in turbulent flows. Center for turbulence Research report. **CTR-S88** (1988) 193-208.

3 M. ANDERSEN: Topology of of streamlines and Vorticity Contours for Two-Dimensional Flows. PhD thesis, Technical University of Denmark, (2013).

4 H. KUDELA AND Z. M. MALECHA: Eruption of a boundary layer induced by a 2D vortex patch. Fluid Dynamics Research, **41 (5)** (2009).

5 A. R. NIELSEN: Dynamics and Topology of Boundary Layer Eruption. M.Sc. thesis, Technical University of Denmark, Department of Applied Mathematics and Computer Science, (2017).

6 M. BRØNS: Streamline topology: Patterns in fluid flows and their bifurcations. Advances in Applied mechanics, **41** (2007) 1-42.